Mathematical modelling of katabatic winds over Schirmacher region, East Antarctica

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Received 25 October 2005; revised 25 September 2006; accepted 15 February 2007

A one-dimensional mathematical model for flow of katabatic winds over Schirmacher region of East Antarctica has been developed. The model is based on momentum and sensible heat transport to the ice slope surface under calm conditions. A relationship of potential air temperature with the height and elevation of the reference state is suggested and used in the model. The model parameters were estimated using the measurements of surface based meteorological parameters and high resolution maps of pressure contours. The wind velocities have been computed using the model on actual terrain slope around 130° in which direction maximum katabatic flow moves towards the periphery of the continent, the ratio of mean bulk coefficients ($C_H/C_M$) and over large variations in slope angle ($\alpha$), potential temperature difference between air parcel and slope surface ($\theta$), and slope length ($l$). The results suggest that the inclination angle of terrain slope and the distance at which inversion forms, control the speed of katabatic winds. At the same time, the direction of katabatic wind is controlled by the slope of the icy terrain.

Keywords: East Antarctica, Katabatic winds, Cold air parcel, Sensible heat, Ice surface, Schirmacher region

PACS No.: 92.60. Gn

1 Introduction

Katabatic winds, caused by radiative cooling of near-surface air are an almost ubiquitous feature of the Antarctic costal slopes. The cooling of the air causes a thermal inversion, resulting in a favourable pressure gradient for the down slope wind component. The inversion air mass on the smooth ice surface starts moving down under the influence of gravity, forming katabatic winds. The katabatic winds are thus unidirectional (flowing out from interior of continent towards the periphery). The strength of katabatic wind is dependent on the slope of ice surface and is extremely strong, persistent and steady in both speed and direction. Various studies have documented the persistence of the drainage flow pattern.

Realizing the importance of katabatic winds over Antarctica, Ball (1956) was the first to describe the basic theory of strong katabatic winds. The main purpose of this model was to theoretically simulate strong katabatic winds with particular reference to the slope around the edges of Antarctica. The model calculated acceleration of air on the slopes, and probably, before reaching the foot, an approximate equilibrium was set up between the frictional resistance and the katabatic force. The problem was simplified by assuming the steady state, taking the upwind supply of cold air and supposing the potential temperature of the katabatic flow to be constant. The Earth’s rotation was neglected. Later on, Ball (1960) modified the above model by including landmasses or icy land gaining heat by radiation during the day and losing heat at night. This resulted in unstable stratification, marked by turbulence and convective activity in the surface layers by day, and stable stratification and suppression of turbulence by night.
The nocturnal inversion resulted both in the frictional decoupling of the surface layers from those above and in local pressure gradients which had little or no relation to the synoptic pressure distribution. Therefore, in general, it was not possible either to deduce the wind direction and speed from the synoptic pressure distribution at night, or to deduce the direction and spacing of the isobars from the nocturnal wind measurements without considerable knowledge of local conditions.

The results obtained by the above methods were compared using hydraulic approach. The authors took a new hydraulic approach in the development of a model, the detailed vertical structure of the flow was replaced by a quiescent stably stratified environment and an equivalent flowing cooling layer, surface stress and interfacial entrainment. A scaling which contained most of the parametric behaviour was found. It was shown that interfacial entrainment was the dominating retardation mechanism of the flow and that surface stress was relatively unimportant. This model was used for understanding the development of katabatic flow with distance and time on ice slopes, the mass of air involved, and the mixing within the flow and between the flow and the ambient air. The properties of such a flow were predicted and its effect on the transport and dispersion of participating air was assessed. These models were only one-dimensional. Nappo and Rao (1987) developed a time dependent 2-dimensional model of pure katabatic flow over a simple finite length slope. This numerical model based on a turbulent kinetic energy closure, accounted for the effects of radiative cooling and turbulent friction at the surface, and entrainment of air aloft from a quiescent stably stratified ambient atmosphere. This model provided information on the temporal and spatial evolution and structure of katabatic flows.

Kondo and Sato (1988) introduced momentum and sensible heat transports even under calm conditions; the concept of a ‘cold air parcel’ was introduced for describing the bulk properties of drainage flow. The mean bulk coefficients depended on the roughness length for the velocity and potential temperature profiles and were decreasing functions of the slope length. However, none of the above models considered bulk coefficients of katabatic flows. With the advent of newer technologies for terrain mapping, the resolution of terrain elevation is being measured with greater accuracy, helping the development of models for local area.

In the present analysis, the basic models of Ball (1956), Kondo and Sato (1988) and Gallee and Pettre (1998) have been adopted for the development of the model for the katabatic winds at Schirmacher region of East Antarctica and the model validation has been done on the basis of experimental results obtained at the Indian Antarctic Research station, Maitri. The model parameters were estimated on the basis of measured values. The model is used to calculate the velocities of katabatic winds over Schirmacher region of east Antarctica.

2 Site description

The Schirmacher region is one of the smallest oases situated over the Droning Maud land and is about 70 km south of Princes Astrid coat of East Antarctica. It is small moraine of the Antarctic glacier with ~ 35 km² area, having a range of low lying hills. The elevation of Schirmacher ranges between 0 and 228 m with an average elevation of about 118 m. It has a maximum length of about 17 km and maximum width of 3.5 km. It is oriented approximately in the east-west direction and forms an obstruction to the flow of the glacier. Towards the north of it is the shelf ice, while towards east is the polar ice. The thickness of polar cape ice goes on increasing towards the pole. The coordinates of the Oases are 70°46′04″-70°44′24″S; 11°49′54″(+/− 48)-11°26′03″ (+/− 02) E. About 3% of Schirmacher Oasis is always free from snow/ice cover, even during the winter. The remaining area is normally covered with snow in winter, which melts off during the summer. Figure 1 shows a map of the Schirmacher Oasis, the location of Indian Antarctic station, Maitri and Russian Antarctic station, Novolazarevskaya are also situated on this oasis. This oasis is on higher elevation, thus the katabatic flow can not come from this direction. It is only from the SE quadrant that the slopes are maximum and it is the direction (around 130° of

![Fig. 1 — Map of Schirmacher Oases where Maitri station is situated](image-url)
3 The model

The concept of a cold air parcel resting on icy slopes was introduced by Kondo and Sato (1988) to develop a mathematical model for drainage flow. As the parcel descends from the top of the slope (crest), its potential temperature decreases due to the transport of sensible heat to the slope surface. The potential temperature of the parcel $\Theta_1$ is defined by:

$$\Theta_1 = \frac{\Theta + \Theta_s}{2} \quad \text{(1)}$$

where $\Theta$ and $\Theta_s$ are the potential temperatures of the ambient atmosphere and the slope surface respectively.

The potential temperature difference ($\Theta$) is assumed to be constant along the slope and described as:

$$\Theta_1 - \Theta_s = \Theta - \Theta_1 = \Theta \quad \text{(2)}$$

The parcel is shown schematically in Fig. 2, in which the slope length $l$ and vertical drop $\delta z = l \sin \alpha$ are measured from the crest, where $\alpha$ is the slope angle. The released heat per unit time and unit area is equivalent to the mean sensible heat transported to the slope surface, given by

$$H_p = \frac{1}{\rho \cdot cp} \left( \frac{1}{l} \int H_0 \, dt \right) \quad \text{(3)}$$

where $c_p$ and $\rho$ are specific heat and density of air, respectively. The angular brackets denotes the mean value over the interval $l$, and $t$ is the travelling time of the parcel. The mean sensible heat is expressed as

$$\langle H_0 \rangle = c_p \rho C_H (u) \Theta \quad \text{(4)}$$

where $C_H$ is the mean bulk coefficient for the heat transport from the parcel to the slope surface. Equations (3) and (4) yield the thickness of the parcel as

$$h = C_H l \quad \text{(5)}$$

where $h$ corresponds to the inversion height, $h_{inv}$ of actual drainage flow.

The equation of motion of the parcel is given by

$$\frac{du}{dt} = h g \frac{\sin \alpha}{\Theta_0} - \frac{\tau_0 + \tau_h}{\rho} \quad \text{(6)}$$

where $\Theta_0$ is the potential air temperature in a reference state. The first term on the right hand side denotes the gravitational force, and $\tau_0$ and $\tau_h$ in the second term denote the frictional forces at the slope surface and at the interface between the parcel and the ambient atmosphere, respectively ($\tau_h$ corresponds to the interfacial drag due to the entrainment).

4 Relation between the air parcel and katabatic flow

Because the present parcel model does not describe the internal structure of drainage flow, the relation between the concept of the parcel and actual drainage flow is examined with the aid of some dynamic models and observations.

The characteristic thickness $h'$ and velocity $u'$ of drainage flow are defined by

$$h'(x) = \frac{1}{\Theta_0} \int_0^x (\Theta(x, z) - \Theta_0) \, dz \quad \text{(7)}$$

$$u'(x) = \frac{1}{h'(x)} \frac{\partial h'(x)}{\partial x} \quad \text{(8)}$$

where $\Theta'$ is the deviation of potential temperature of drainage flow from ambient potential temperature, and $u'$ the velocity of the drainage flow itself. The origin of the coordinate system is located at the crest.
The x-axis is taken in the down slope direction and the z-axis normal to the slope. It is assumed that \( \theta \) defined by Eq. (2) is constant along the slope. For the steady state, drainage flow is governed by the following equations:

\[
\begin{align*}
\frac{u \partial u}{\partial x} + \frac{w \partial u}{\partial z} &= \frac{\partial}{\partial z} \left( \frac{\tau \sin \alpha}{\rho \Theta_0} \right) - g \theta \sin \alpha \\
\frac{u \partial \theta}{\partial x} + \frac{w \partial \theta}{\partial z} &= - \frac{H}{\rho c_p} \gamma u \sin \alpha
\end{align*}
\]

where \( \tau \) and \( H \) are downward momentum and sensible heat fluxes, respectively. After integrating these equations vertically and transforming them, expression for velocity can be derived.

5 Analytical solution of characteristic velocity \( \dot{v} \) of katabatic wind

From Eqs (9) and (11)

\[
\frac{u \partial \dot{v}}{\partial x} + \frac{w \partial \dot{v}}{\partial z} = \frac{\partial}{\partial z} \left( \frac{\tau \sin \alpha}{\rho \Theta_0} \right) - g \theta \sin \alpha
\]

After integrating over \( z \) from zero to infinity, Eq. (12) results in

\[
\int_0^\infty \frac{u \partial \dot{v}}{\partial x} dz = - \frac{\tau_0}{\rho} + \frac{\partial \theta}{\partial z} \sin \alpha
\]

where, \( \tau = \tau_0 \) at \( z = 0 \) and \( \tau = 0 \) at \( z = \infty \) and \( \tau_0 \) denotes the momentum heat flux at the surface. With the profile factors \( a_1 \) and \( a_3 \), Prandtl (1952), Eq. (13) is rewritten as

\[
\frac{5}{2} \frac{\partial \theta}{\partial x} = - \frac{\tau_0}{\rho} + \frac{\partial \theta}{\partial z} \sin \alpha
\]

where

\[
\frac{\tau_0}{\rho} = C_M \dot{\theta}_0
\]

Substitution of Eq. (15) into Eq. (14) yields

\[
\dot{h} \frac{\partial \dot{\theta}_0}{\partial x} = \frac{2}{5} \frac{\partial \theta}{\partial z} \sin \alpha - \frac{\dot{\theta}_0}{C_M} + \frac{5}{2} \frac{\dot{h}}{h \dot{\theta}_0} \frac{\partial \theta}{\partial x}
\]

Considering that \( \partial \dot{\theta}/\partial x = 2 \dot{\theta} \partial \dot{\theta}/\partial x = 2 \partial \dot{\theta}/\partial r \), Eq. (16) corresponds to equation of motion of the parcel, Eq. (6). The shapes of the potential temperature deviation and the velocity distributions cause the factor 2/5. Instead of the frictional force at the interface between the parcel and the ambient atmosphere, the drag due to the change in characteristic thickness appears in Eq. (16). This term has the same effect as the drag due to entrainment. An analytical solution of Eq. (16) for \( \dot{\theta} \) at \( x \leq l \) by considering \( \dot{\theta} = 0 \) at \( x = 0 \) and constant \( C_H \) and \( C_M \) is given as

\[
\dot{h} \frac{\partial \dot{\theta}_0}{\partial x} = \frac{3}{5} \frac{\partial \theta}{\partial z} \sin \alpha - \frac{\dot{\theta}_0}{C_M} + \frac{1}{3 C_M} \frac{\partial \theta}{\partial x}
\]

Kondo and Sato (1988) did not explain the relationship of potential air temperature with the height and elevation of the reference state. In the present paper, following relationship of potential air temperature with the height and elevation of the reference state is developed:

\[
\Theta_0 = T_{Maitri} - 9.8 \frac{h - 118}{1000} \frac{\dot{\theta}_0}{\dot{\theta}}
\]

where, \( T_{Maitri} \) is temperature in °C, \( h \), height in metre corresponding to slope length and 118 m is the elevation of Maitri station.

For calculating the katabatic wind velocity using the above model, following data set was used:

(i) Surface based meteorological parameters, e.g. wind velocity and direction reported by an automatic weather station, as well as, the 3-hourly measurements done by the India Meteorological Department, New Delhi.

(ii) Temperature profiles recorded by Radiosonde flights from the Russian station, Novolazarevskaya, which is situated only 3 km away towards SE of the Maitri station. Radiosonde profiles also provide temperature gradient across surface based and elevated layers.
(iii) At the same time, some supporting observations like mslp charts, cloud cover, etc. were downloaded from the various websites on the internet.

On the basis of the measurements made at the Schirmacher region, the parameters are estimated and listed in Table 1.

### Table 1 — Estimated parameters at the Maitri station

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Slope angle $\alpha$</td>
<td>0.0010 deg (for a distance of 100 km from the Maitri station); 0.0012 deg (for 500 km); 0.0014 deg (for 1000 km); 0.0015 deg (for 2000 km).</td>
</tr>
<tr>
<td>2. Potential temperature difference between ambient atmosphere and parcel or slope surface</td>
<td>5°C to 20°C depending on the inversion strength</td>
</tr>
<tr>
<td>3. Slope length</td>
<td>100-2000 km</td>
</tr>
<tr>
<td>4. Ratio of bulk coefficients $C_H/C_M$ (Kondo and Sato, 1988)</td>
<td>1.00</td>
</tr>
<tr>
<td>5. Temp. variations at Maitri, Antarctica</td>
<td>0-30°C</td>
</tr>
</tbody>
</table>

### 6 Results

The katabatic wind velocity along with slope length is computed using parameters mentioned in Table 1 for various slope angles and for fixed potential temperature difference and the reference temperature as 0°C. The computed results, given in Fig. 3, shows that the wind velocity increases with distance as well as the slope angle. The same exercise has been repeated by varying potential temperature difference as 15 and 25°C [Fig. 4 (a)-(b)]. However, there is not

![Fig. 3 — Variation of wind velocity with distance at different slope angles with constant potential temperature difference and reference temperature](image1)

![Fig. 4 — Variation of wind velocity with distance at different slope angles with potential temperature difference and reference temperature values of (a) 25°C, −10 °C and (b) 15°C, 0°C respectively](image2)
much change in the velocity with distance, even though there is a change in the gradient. Similarly, there seems to be no appreciable change in the computed wind velocity with change in the reference level temperature, as shown in Fig. 5. In the mid-day, particularly in the summer season, the atmosphere over the oasis is determined by the thermal convection, which gets dissipated or gets eroded by sudden onset of katabatic flow, in which wind velocity is slightly more than 6 m/s.

In this situation, the formation of inversion really takes place in the near vicinity of the Schirmacher oasis and this inversion air mass is pulled down by gravity, as shown in Fig. 6(a)-(d). In these figures the slope distance and the potential temperature difference have been varied with constant potential temperature difference and reference temperature.

The actual slope of around 130 deg has been utilized to calculate the velocities. It is extremely important to note that the Schirmacher Oasis being rocky, will take more time to cool and the icy surfaces around will cool much faster, leading to the formation of katabatic winds in the evening, which continues for the whole night. Of course in winter, this day and night demarcation vanishes.

6.1 Comparison with previous theoretical works

Theoretically developed model for katabatic winds flow has been compared with previous theoretical works by Ball (1956), who described the basic theory of strong katabatic winds as described earlier. Kondo and Sato (1988) introduced momentum and sensible heat transport to the slope surface even under calm conditions (also described earlier). However, none of the above models considered mean bulk coefficients of katabatic flows and calculation for potential air temperature in reference state.

In the present katabatic wind model a relationship [Eq. (18)] of potential air temperature with the height and elevation of reference state (Maitri station) was developed on the basis of the theory of dry adiabatic lapse rate and used in the model. The main focus of the authors was to develop the model for Maitri station, which is situated over Schirmacher region. Keeping this in mind, the model is developed to compute wind velocities on actual terrain slope of around 130°, because maximum winds come from this direction. Apart from this, ratio of mean bulk coefficients $C_H/C_M = 1$ was taken as fixed in the model, as roughness of ice surface is very less. This model developed particularly for Maitri station gives more accurate results as compare to previous existing models.

6.2 Comparison with in-situ measurements

Model calculations for comparison with in situ measurement cases for 14 and 15 Mar. 2003 and 14 Sep. 2003 were obtained as shown in Figs 7, 8 and 9, respectively, at Maitri, Antarctica. The comparison has been made by varying slope angle with a constant potential temperature difference of 15°C and reference temperature of 0°C [(Fig. 7(a)-(b)], and similarly by varying reference temperature with constant potential temperature difference = 5°C and slope angle = 0.019° [Fig. 8(a)-(b)].

It is found that velocity increases with distance as well as slope angle but there is not much change in the velocity with distance, even though there is a change in potential temperature difference with constant reference temperature = 0°C and slope angle = 0.012° [Fig. 9(a)-(b)]. Similarly, there seems to be no appreciable change in the calculated and measured velocities with change in the reference level temperature.
The comparison of calculated and measured velocity with distance has found them to be in good agreement, with aforesaid parameters and conditions. The inversion air mass actually starts rolling from the nearby regions, as the icy surfaces become cooler much before that at the Schirmacher Oasis. In the late night or in winters, the entire area is cooler, thus severe katabatic winds blow for a number of hours or

Fig. 6 — Variation of wind velocity with distance at different slope angles with constant potential temperature difference and reference temperature (a) 0°C, (b) –30°C, (c) 0°C and (d) –30°C
even days. The vital role is played seemingly not by the gradient of inversion but by the angle of terrain or the slope angle.

7 Conclusion
A one-dimensional mathematical model for katabatic wind flow over Schirmacher Oasis region has been developed. The model is based on momentum and sensible heat transport to the ice slope surface under calm conditions. A relationship of potential air temperature with the height and elevation of the reference state is developed on the basis of the theory of dry adiabatic lapse rate and used in the model. For the computations of katabatic wind speeds, measurements made by the monostatic acoustic sounder, radiosonde, surface based meteorological parameters have been utilized. The katabatic wind velocities have been computed over a large variation...
in slope angle ($\alpha$), potential temperature difference between air parcel and slope surface or difference between ambient atmosphere and air parcel ($\theta$), slope length ($l$) and the ratio of mean bulk coefficient ($C_H/C_M$). Results obtained suggest that the inclination angle or terrain slope and the distance at which inversion forms, control the speed of katabatic winds.

At the same time, the direction of katabatic wind is controlled by the slope of the icy terrain.

Acknowledgements

The authors are grateful to CSIR, New Delhi for financial support. Dr. Victor Lagun from Arctic and Antarctic Research Institute, St. Petersburg, Russia is thankfully acknowledged for providing the Novolazarevskaya data and useful discussion. The authors express their sincere thanks to Dr. Svetlana Jagovskina for her encouraging comments on the modelling work. Thanks are also due to India Meteorological Department, New Delhi for providing surface data.

References