Cumulus convection and lateral boundary conditions in a limited area model

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Abstract. Three versions of Kuo’s cumulus parameterization have been tested in a limited area model to investigate their comparative performances. Results show that the version of Anthes produces better forecasts than those produced by other versions. To identify a suitable scheme of lateral boundary conditions for the limited area model, impact of two time-invariant and two time-dependent boundary conditions have been examined. The forecasts suggest that the time-dependent tendency modification scheme, based on large-scale tendencies obtained from observed data, is a better boundary scheme for the model. Furthermore, the forecast produced with the revised version of the model incorporating improved versions of Kuo’s scheme and lateral boundary conditions shows an overall improvement.

Keywords. Limited area model; cumulus convection; lateral boundary conditions; sponge damping; Kuo’s cumulus parameterization.

1. Introduction

Cumulus convection is considered to be one of the important facets of numerical modelling for short and medium range weather prediction, particularly over the monsoon region. Another important component of modelling which is peculiar to limited area models, is the specification of lateral boundary conditions.

In the present paper, an attempt was made to study different versions of Kuo’s scheme and lateral boundary conditions in a limited area model (Singh et al 1990) to identify suitable schemes of convection and boundary conditions. Comparatively better schemes that emerged from the above investigation are incorporated in the model to produce 48 h forecasts.

The paper presents different versions of Kuo’s scheme and lateral boundary conditions. The forecast produced with the improved version of the model is discussed.

2. Description of the model

The model is a regional six-level primitive equation model in \( \sigma \)-coordinates. The model is staggered in both vertical and horizontal directions. Arakawa’s B-type of staggering was used in the horizontal direction. The six sigma levels were close to 950, 850, 700, 500, 300 and 150 hPa. The top of the model was fixed at 100 hPa. The horizontal domain extends from \( 10^\circ \)S to \( 40^\circ \)N and \( 50^\circ \)E to \( 120^\circ \)E. The horizontal domain had \( 45 \times 33 \) grid points on a Mercator projection with 150 km as grid length.

The lateral boundary conditions are given in §5. In the vertical, the boundary
condition $\sigma = 0$ at $\sigma = 0$ and $\sigma = 1$ was used. Mass, energy, potential temperature and the variance of potential temperature conserving schemes (Arakawa and Mintz 1974; Arakawa and Lamb 1977) for space derivatives were used. The horizontal advection terms in the momentum equations were computed with fourth-order accuracy finite difference scheme (Arakawa's 13-point Jacobian scheme). The other terms were computed with a second-order accuracy finite difference scheme. For time integration, a leap frog scheme with Asselin (1972) time filter was used with a time-step of 3 minutes. The physical processes incorporated in the model are: large-scale condensation, dry convective adjustment, horizontal and vertical diffusion, sensible heat supply and evaporation over the sea. Three versions of convection were incorporated in the model. A detailed description of the model may be found in Singh et al (1990).

3. Data

The FGGE IIIb grid point data of 12 GMT 7 July 1979 were used as input to the model for all numerical experimentations. The dominant synoptic feature was an intense monsoon depression centred over the head Bay of Bengal on 7 July. This system moved initially westward and subsequently northwestward and dissipated on 9 July.

4. Study on cumulus convection

The cumulus parameterization scheme of Kuo (1965, 1974) has been widely used in large-scale numerical weather prediction models due to its simple formulation and computational economy. Several investigators showed that the classical scheme of Kuo (1965) underestimated rainfall rates. Furthermore, it was noted that there was a disproportionate partitioning of the available moisture supply into moistening and heating. Kuo (1974) recognized the shortcomings of his earlier scheme and introduced a more realistic division of the moisture by bringing in a moistening parameter $b$. Considerable attention has been focussed on improving the performance of Kuo's (1974) scheme by trying to find the most appropriate value of the moistening parameter $b$. In the following section, an outline of the Kuo's scheme is given.

4.1 An outline of the Kuo's scheme

In the Kuo's scheme of cumulus parameterization the heating and vertical flux of moisture is invoked only if the sounding is conditionally unstable and the net convergence of moisture is positive. Following Kanamitsu et al (1983), the large scale supply of moisture (net convergence of moisture) $I_L$ is given by

$$I_L = \int_{\eta_s}^{\eta_r} (\hat{\epsilon} q/\hat{\epsilon} t) d\sigma.$$  \hspace{1cm} (1)

Krishnamurti et al (1983a) introduced an additional source of moisture supply $\eta I_L$ which is a non-measurable mesoscale (or sub-grid scale) supply. The term $\eta I_L$ was introduced to provide sufficient moisture supply in the vertical column to account
for the observed rainfall rates and moistening. The total moisture supply was expressed by

\[ I = I_L (1 + \eta). \]  

(2)

\( \eta \) is known as the mesoscale convergence parameter. Following Kuo (1965), the maximum amount of moisture supply required to produce a grid square cloud was expressed by

\[ Q = - \int_{q_a}^{q_r} (q_a - q) \, d\sigma - \frac{C_F}{L} \int_{q_a}^{q_r} (T_s - T) \, d\sigma. \]  

(3)

The first term on the right side is denoted as \( Q_a \) and the second term as \( Q_b \). The terms \( Q_a \) and \( Q_b \) respectively denote the supply needed for moistening and heating of a unit column. Introduction of a parameter \( b \) following Kuo (1974), the rainfall rate \( R \) and moistening rate \( M \) may be expressed by

\[ R = I_L (1 + \eta)(1 - b), \]  

(4)

\[ M = I_L (1 + \eta)b. \]  

(5)

Following Kuo (1974), the two additional parameters \( a_b \) and \( a_q \) may be expressed by

\[ a_b = \frac{I(1 - b)}{Q_b} = \frac{I_L (1 + \eta)(1 - b)}{Q_b}, \]  

(6)

\[ a_q = \frac{Ib}{Q_q} = \frac{I_L (1 + \eta)b}{Q_q}. \]  

(7)

In this formulation there are two unknown parameters \( b \) and \( \eta \) that are to be determined.

The heating and moistening by clouds are proportional to the temperature and humidity difference between the model cloud and the environment. Consequently the heating and moistening at each level are expressed as

\[ \frac{\partial T}{\partial t} = a_b (T_s - T), \]  

(8)

\[ \frac{\partial q}{\partial t} = a_q (q_s - q). \]  

(9)

From (6) and (7), \( a_b \) and \( a_q \) are determined if \( b \) and \( \eta \) are known and thus the parameterization is closed.

In the following section we shall show how \( b \) and \( \eta \) are determined by different versions of Kuo's scheme.

4.2 Three versions of Kuo's scheme

(i) Kuo (1974) version (KU74): This is essentially the earlier Kuo (1974) scheme of parameterization. The parameters \( b \) and \( \eta \) are set zero. The coefficients \( a_b \) and \( a_q \) are determined as follows:

\[ a_b = I_L / Q_b, \]  

(10)

\[ a_q = 0. \]  

(11)
The above formulation for computation of $a_\theta$ and $a_q$ was used by Krishnamurti et al (1980). This version will hereafter be referred to as KU74.

(ii) Anthes (1977) version (AN77): The parameter $\eta$ is set zero. The parameter $b$ is computed following Anthes (1977).

$$b = \begin{cases} 
\left( \frac{1 - \langle RH \rangle}{1 - RH_c} \right)^n & \text{when } \langle RH \rangle > RH_c \\
1 & \text{when } \langle RH \rangle \leq RH_c
\end{cases}$$

(12)

where $\langle RH \rangle$ is the mean relative humidity in the cloud layer and $RH_c$ is the critical relative humidity below which there is only moistening. The values, $RH_c = 0.5$ and $n = 3$ are used in the present study.

The heating and moistening rates were calculated following the usual Kuo (1974) scheme. The coefficients $a_\theta$ and $a_q$ are determined by

$$a_\theta = (1 - b)I_L/Q_\theta,$$

$$a_q = b L_q/Q_q.$$  

(13)

(14)

This version will hereafter be referred to as AN77.

(iii) Krishnamurti et al (1983) version (KR83): Krishnamurti et al (1983a) proposed a closure for $b$ and $\eta$ by a screening multiple-regression analysis of GATE observations. The normalized heating and moistening $R/I_L$ and $M/I_L$ were regressed against a number of large-scale variables. Based on GATE observations, significant correlations for moistening and heating were noted by them from the following relations,

$$M/I_L = a_1 \zeta + b_1 \bar{\omega} + c_1,$$

$$R/I_L = a_2 \zeta + b_2 \bar{\omega} + c_2.$$  

(15)

(16)

where $\zeta$ is the relative vorticity at 700 hPa and $\bar{\omega}$ is the vertically-averaged vertical $p$-velocity. $a_1$, $a_2$, $b_1$, $b_2$, $c_1$ and $c_2$ are coefficients of regression. Magnitudes of these coefficients are given by Arunkumar (1989). Expressions for $b$ and $\eta$ can be obtained from (4), (5), (15) and (16).

$$b = \frac{a_1 \zeta + b_1 \bar{\omega} + c_1}{(a_1 + a_2) \zeta + (b_1 + b_2) \bar{\omega} + (c_1 + c_2)},$$

$$\eta = (a_1 + a_2) \zeta + (b_1 + b_2) \bar{\omega} + (c_1 + c_2) - 1.$$  

(17)

(18)

Thus the values of $b$ and $\eta$ could be determined with model values of $\zeta$ and $\bar{\omega}$. The coefficients $a_\theta$ and $a_q$ were then obtained by

$$a_\theta = I_L (1 + \eta)(1 - b)/Q_\theta,$$

$$a_q = I_L (1 + \eta)b/Q_q.$$  

(19)

(20)

The scheme will be referred to as KR83.

4.3 Numerical experiments

Two numerical experiments using the three versions outlined in the previous section were made. In the first experiment (EX1), following Kanamitsu et al (1983) the
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conditions to be satisfied for invoking convection are: (i) conditionally unstable atmosphere, (ii) positive moisture convergence in the cloud layer and (iii) environmental mean relative humidity in the cloud layer exceeds a critical value of 80%. In the second experiment (EX2) convection is invoked if conditions (i) and (ii) of EX1 are satisfied. No constraint is imposed on mean relative humidity.

In both the experiments the time-invariant lateral boundary conditions were used.

4.4 Discussion

In both experiments the model was integrated up to 48 h using input data of 12 GMT 7 July 1979.

In the first experiment (EX1) the forecast results of KU74, AN77 and KR83 were found very similar. In the second experiment (EX2) both KU74 and KR83 schemes produced spurious results. Hence, the forecast results of KU74 of EX1 (EX1/KU74) and AN77 of EX2 (EX2/AN77) are discussed below for comparison.

(i) 850 hPa wind: Figure 1 shows 48 h predicted 850 hPa winds of EX1/KU74 and EX2/AN77 schemes. The observed winds of 12 GMT 9 July 1979 are also shown in the same figure for comparison. It can be seen that in both the experiments the circulation features are reasonably well-predicted, particularly the cyclonic circulations associated with the depression, circulations around the equator and the cross-equatorial flow. However, the easterlies to the north and westerlies to the south of the depression are found to be weaker. It can also be seen that the westerlies off the west coast are overpredicted by 10–15 kt.

(ii) Rainfall: The predicted rainfall rates (12–36 h) of EX1/KU74 and EX2/AN77 are shown in figure 2. The observed rainfall during the 24 h period ending at 00 GMT 9 July 1979 (Source: Krishnamurti et al 1983b) are also shown in the same figure. In general, the areal distribution of rainfall is predicted better in EX2/AN77. The predicted rainfall rates associated with the depression are underpredicted in both the experiments. Grell et al (1991), in their semiprognostic tests in middle latitude with three versions of Kuo’s scheme noticed underprediction of heating and moistening rates. They noted the need to include mesoscale moisture convergence to correctly predict the heating and moistening rates in large-scale models.

In EX1/KU74 the rainy area associated with the depression has two maxima, one of 20-6 mm at 20°N, 85°E and the other of 10·2 mm at 20°N, 76°E. The observed rainfall maximum associated with the depression is 50 mm at 22°N, 80°E. However, in EX2/AN77 the rainy area associated with the depression shows three rainfall maxima one of 12·4 mm at 18°N, 83°E, other of 10·2 mm at 22°N, 78°E and another of 24·4 mm at 20°N, 75°E.

(iii) Track of the depression: The predicted track of the depression of EX1/KU74 and EX2/AN77 is shown in figure 3. For comparison, the observed track of the depression is also shown in the figure. In can be seen that in both EX1/KU74 and EX2/AN77 the predicted tracks are to the north of the observed track and the movement is slower in the case of EX2/AN77 in the first 24 h. In the next 24 h the depression moved westward in EX2/AN77 while in EX1/KU74 the movement is towards southwest. The 48 h forecast position of the depression is slightly to the south of the actual in EX2/AN77 while it is located far to the south-east in EX1/KU74. Thus, the westward movement of the depression in a 24–48 h period is better predicted in EX2/AN77.
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Figure 1c

Figure 1. 850 hPa wind (a) 48 h forecast from EX1/KU74, (b) 48 h forecast from EX2/AN77 and (c) Observed at 12 GMT 9 July 1979.
Figure 2. Rainfall (mm) (a) Forecast from EX1/KU74, (b) Forecast from EX2/AN77 and (c) Observed for the 24 h period ending 00 GMT 9 July 1979 (dots denote rainfall < 5 mm in (a) and (b) and rainy area in (c)).
(iv) Phase speed and positional error: The predicted and observed phase speed of the monsoon depression and the positional errors are shown in Table 1. It can be seen that the predicted phase speed of the depression in the first 24 h of EX1/KU74 is comparable to the observed phase speed while the predicted phase speed of EX2/AN77 is less than the observed. In the next 24 h the predicted phase speed of EX1/KU74 is less than half of that observed while the phase speed of EX2/AN77 is comparable to the observed.

The positional error is the same for both EX1/KU74 and EX2/AN77 in the 24 h forecast. In the 48 h forecast the positional error is much higher in the case of EX1/KU74 scheme.

(v) RMS error: The root mean square (RMS) errors for $u$ and $v$ components of wind and geopotential height ($Z$) at 850, 500 and 200 hPa for both EX1/KU74 and EX2/AN77 are shown in Table 2. It can be seen that the RMS errors of $u$, $v$ and $Z$ are slightly less for EX2/AN77 at all the three levels in both 24 and 48 h forecasts. Slingo et al. (1988), in their study of impact of various modified physical processes in
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Table 2. 24 and 48 h RMS errors of $u$, $v$ (ms$^{-1}$) and $Z$ (m) at 850, 500 and 200 hPa levels for experiments EX1/KU74 and EX2/AN77.

<table>
<thead>
<tr>
<th>Level (hPa)</th>
<th>Variable</th>
<th>24 h</th>
<th>48 h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>EX1/KU74</td>
<td>EX2/AN77</td>
</tr>
<tr>
<td>850</td>
<td>$u$</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>40.8</td>
<td>40.6</td>
</tr>
<tr>
<td>500</td>
<td>$u$</td>
<td>3.1</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>2.9</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>47.1</td>
<td>45.0</td>
</tr>
<tr>
<td>200</td>
<td>$u$</td>
<td>5.0</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>4.8</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>73.3</td>
<td>71.6</td>
</tr>
</tbody>
</table>

the ECMWF model, showed that improvements in RMS wind errors at 850 hPa could be found only after day 2 forecast.

From the above discussions on wind field, rainfall, track of the depression, phase speed, positional error and RMS error it can be inferred that EX2/AN77 is a better scheme for cumulus parameterization.

5. Study of lateral boundary conditions

The treatment of lateral boundaries is an important problem for regional NWP models. Over the years, a number of techniques for treating lateral boundaries have been tested. Sundstrom and Elvius (1979) and Davies (1983) have presented excellent reviews of commonly used lateral boundary conditions for regional models. In this study two time-invariant and two time-dependent lateral boundary conditions have been applied to examine their impact on model forecasts. In all experiments the EX1/KU74 version of convection has been incorporated.

5.1 Four schemes for lateral boundary conditions

(i) Fixed boundary conditions without sponge damping ($B_1$): In this, all the prognostic variables at the boundary were kept fixed throughout the period of integration i.e. the initial values were not modified at the boundary. This scheme will hereafter be referred to as the $B_1$ scheme.

(ii) Fixed boundary conditions with sponge damping ($B_2$): The sponge damping scheme of Perkey and Kreitzberg (1976) was adopted. Following their scheme the prediction of any dependent variable $X$ can be written as

$$X_n(l) = X_p(l) + W(l) \left[ \frac{\partial X_m}{\partial t} \right]_l \Delta t,$$

(21)

where subscripts $n$ and $p$ denote the new value after the boundary condition is applied and a previous value at a former time and $m$ is the model-computed tendency.
Figure 4. Predicted track of the monsoon depression at 850 hPa level for (a) $B_1$ and $B_2$ schemes and (b) $B_3$ and $B_4$ schemes.

Table 4. Predicted and observed phase speeds of the monsoon depression at 850 hPa.

<table>
<thead>
<tr>
<th>$T$ (h)</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–24</td>
<td>570</td>
<td>410</td>
<td>250</td>
<td>240</td>
<td>560</td>
</tr>
<tr>
<td>24–48</td>
<td>200</td>
<td>325</td>
<td>400</td>
<td>420</td>
<td>420</td>
</tr>
</tbody>
</table>
Figure 5. 48 h accumulated rainfall (mm) (a) $B_1$ scheme, (b) $B_2$ scheme, (c) $B_3$ scheme and (d) $B_4$ scheme.
The results presented in §5 reveal that the time-dependent tendency modification scheme \( B_4 \) is a superior lateral boundary condition for the model.

6. Prediction with revised version of the model

The study on cumulus convection discussed in §4 reveals that the Anthes (1977) scheme in Experiment 2 (EX2/AN77) scheme produced better forecasts. Similarly, from the study on lateral boundary conditions discussed in §5 it was found that the time-dependent tendency modification scheme \( B_4 \), based on large-scale tendencies obtained from observed data produced better forecasts. A prediction experiment was carried out with the revised version of the model incorporating the EX2/AN77 scheme of convection and \( B_4 \) scheme for lateral boundary conditions.

6.1 Forecast results

The revised version of the model was integrated up to 48 h using input data of 7 July 1979. As the result has been discussed in detail in §§4 and 5, the salient features of the forecasts are presented below.

(i) Track of the depression: The predicted and observed tracks of the monsoon depression are shown in figure 6. It can be seen that the predicted track of the depression is quite close to that observed although the speed of movement was slower. Compared to the tracks in the previous experiments, this was the best predicted track.

![Figure 6. Predicted track of the monsoon depression at 850 hPa level with revised version of the model.](image-url)
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Table 5. 24 and 48 h RMS errors of $u$, $v$ (ms$^{-1}$) and $Z$ (m) at 850, 500 and 200 hPa levels with revised version of the model.

<table>
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<th>48 h</th>
</tr>
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<td>$u$</td>
<td>3.1</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>2.9</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>25.9</td>
<td>27.6</td>
</tr>
<tr>
<td>500</td>
<td>$u$</td>
<td>2.2</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>2.0</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>29.1</td>
<td>34.8</td>
</tr>
<tr>
<td>200</td>
<td>$u$</td>
<td>4.5</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>3.9</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>47.9</td>
<td>64.5</td>
</tr>
</tbody>
</table>

Figure 7. Predicted rainfall (mm) for the period $T = 12-36$ h with revised of the model.

(ii) RMS errors: The RMS errors of $u$ and $v$ components of wind and geopotential height at 850, 500 and 200 hPa levels are shown in table 5. In general, the RMS errors of all the three variables are lower at all levels compared to other experiments reported in this paper.

(iii) Rainfall: The predicted rainfall rates (12–36 h) are shown in figure 7. It can be seen that the areal distribution of rainfall is predicted well. In this case there are two rainfall maxima, one near 84°E, 19°N of 10 mm and another of 21.2 mm near 73.5°E, and 20°N. The spurious rainfall amounts near the eastern boundary are reduced.
7. Conclusions

Based on a case study of numerical prediction of one depression we reach the following conclusions. The study on cumulus convection revealed that inclusion of the Kuo (1974) scheme with Anthes (1977) criteria for computing the moistening parameter $b$ produced better forecasts. The study also suggests that there should be a check on the environmental mean relative humidity of the cloud layer for invoking convection in the model so as to avoid spurious forecasts. The study on lateral boundary conditions suggests that the time-dependent tendency modification scheme of Perkey and Kreitzberg (1976) based on the large-scale tendencies from 12-hourly real data analyses was a better scheme. This result emphasizes the importance of using global models to provide lateral boundary conditions for operational regional models. Furthermore, the revised version of the model incorporating an improved version of convection and tendency modification scheme for lateral boundary conditions produced overall improved forecasts.

Acknowledgements

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